(21213) Roll No. ....

B. Sc. (C.S.)-III Sem.

NP-3602

B. Sc. (Computer Science) Examination, Dec. 2013

DISCRETE STRUCTURES

(BCS-301)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from each Section as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the *five* questions of this Section. Each question carries 3 marks. Very short answer is required. $3 \times 5 = 15$

1. Show that the set of all bit strings is countable. 3

2. Prove that the set {0, 1, 2, 3, 4} is a finite abelian group of order 5 under addition modulo 5 as composition.

Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = x - 1$ and $g(x) = x^2 + 1$, find :

(i) $(fog)(2)$

(ii) $(gog)(2)$. 3

4. When is a simple graph G bipartite? Given an example. 3

5. Show that the proposition $p \to q$ and $\neg p \vee q$ are logically equivalent. 3

Section-B

(Short Answer Questions)

This Section contains three questions, attempt any *two* questions. Each question carries 7½ marks. 7½×2=15

6. Show that $\forall x(P(x) \vee Q(x)) \Rightarrow (\forall x P(x)) \vee (\forall x Q(x))$ by indirect method of proof. 7½

7. Prove that $\sqrt{2}$ is irrational by giving a proof using contradiction. 7½

8. Solve the recurrence relation : 7½

$$a_{n+1} - a_n = 3n^2 - n,\ n \geq 0,\ a_0 = 3.$$

Section-C

(Detailed Answer Questions)

This Section contains five questions, attempt any *three* questions. Each question carries 15 marks. $15 \times 3 = 45$

9. (a) Use mathematical induction to show that : 7

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\ldots+\frac{1}{\sqrt{n}}>\sqrt{n};\ n\geq 2.$$

(b) State the pigeonhole principle. If any 51 integers are chosen from the set {1, 2, 3, ..., 100} then show that among the choosen integers there exist two integers such that one is multiple of the other. 8

10. (a) Let, $(S, \bullet)$ be a semigroup. Then prove that there exists a homomorphism $g: S \to S^S$, where $\langle S^S, 0\rangle$ is a semigroup of functions from S to S under the operation of (left) composition. 8

(b) Prove that every finite group of order n is isomorphic to a permutation group of order n. 7

11. (a) Prove that DeMorgan's laws hold good for a complemented distributive lattice $\langle L, \wedge, \vee\rangle$, viz $(a\vee b)' = a'\wedge b'$ and $(a\wedge b)' = a'\vee b'$. 7

(b) In any boolean algebra, show that : 8

$$(a+b)\ (b+c)\ (c+a) = ab+bc+ca$$

12. (a) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is : 8

$$\frac{(n-k)(n-k+1)}{2}.$$

(b) Prove that a graph is bipartite iff all its circuits are of even length. 7

13. Define with example any five of the following : 3×5

(a) Complete graph

(b) Abelian group

(c) Lattice

(d) Platonic graph

(e) Petersen graph

(f) Biconditional statement

(g) Isomorphism between two algebraic systems.