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(20422)   Roll No.

M.A. - I Year

4185

M.A. Ist Year (Pvt.) (Annual)

Examination, 2022

MATHEMATICS - V I (C)

(Mathematical Statistics)

(G-158)

Time : Three Hours]   *[Maximum Marks :75*

Note : Attempt **five** questions in all. Question **No. 1** is **compulsory**. All questions carry equal marks. Scientific calculator is allowed.

1. (i) If $P(A \cap B) = \frac{1}{2}$,

   $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$ and $P(A) = P(B) = p$

   then find the value of p.   3

   (ii) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.   3

   (iii) For a certain normal distribution the first moment about 10 is 40 and the fourth moment about 50 is 48. What is the arithmetic mean and standard deviation of the distribution.   3

   (iv) If $R_{1.23} = 1$, prove that $r_{2.13}$ is also equal to 1. If $R_{1.23} = 0$, does it necessarily mean that $R_{2.13}$ is also zero?   3

   (v) $x_1, x_2, \ldots\ldots, x_n$ is a random sample from a normal population $N(\mu, 1)$. show that   3

   $$t = \frac{1}{n} \sum_{i=1}^{n} x_i^2$$

   is an unbiased estimator of $\mu^2 + 1$.

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2. (a) From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and is found to be white. What is the probability that out of four balls transferred 3 are white and 1 is black? 7

(b) Let X be a continuous random variable with p.d.f.

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3a & 2 \le x \le 3 \\ 0 & \text{eleswhere} \end{cases}$$ 8

(i) Determine the Constant a,

(ii) Compute $P(x \le 1.5)$.

3. (a) Find moment generating function of binomial distribution and hence find mean, variance, β_1. 8

(b) Establish the recurrence relation for moments of Poisson distribution

$$\mu_{r+1} = r\lambda \mu_{r-1} + \lambda \frac{d\mu_r}{d\lambda} \qquad 7$$

4. (a) If x, y are independent normal variates with mean 6, 7 and variances 9, 16 respectively determine λ such that

$P(2x + y \le \lambda) = P(4x - 3y \ge 4\lambda)$. 7

(b) Define Beta distribution of first kind. Find μ'_r. Hence find mean and variance. 8

5. (a) The variables X and Y are connected by the equation $ax + by + c = 0$ show that the correlation between them is -1 if the sign of a and b are alike and $+1$ if they are different. 7

(b) In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible. 8

Variance of x=a, Regression equations :

$$8x-10y+66 = 0$$

$$40x -18y= 214$$

what are (i) The mean values X and Y.

(ii) The correlation coefficient between X and Y and

(iii) The standard deviation of Y?

6. (a) Show that (i) $R_{1.23} \geq r_{12}$ 7

(ii) $R_{1.23}^2 = r_{12}^2 + r_{13}^2$ if $r_{23}=0$.

(b) From the data relating to the yield of dry bark (x_1) height (x_2) and girth (x_3) for 18 cinchona plants, the following correlation coefficients

were obtained

$$r_{12} = 0.77, r_{13} = 0.72 \text{ and } r_{23}=0.52$$

Find the partial correlation coefficient $r_{12.3}$. and multiple correlation coefficient $R_{1.23}$. 8

7. (a) In two large populations, there are 30 and 25 percent respectively of blue-eyed people. Is this difference likely to be hidden in samples of 1,200 and 900 respectively from the two populations? 8

(b) A random sample of 27 pairs of observations from a normal population gave a correlation coefficient of 0.6. Is this significant of correlation in the population? 7

$[t_{0.05}$ for 25d.f. is 2.06]

8. 1,072 college students were classified according to their intelligence and economic conditions. Test whether there is any association between intelligence and economic conditions. 15

Intelligence

		Excellent	Good	Mediocre	Dull
Economic conditions	Good	48	199	181	82
	Not good	81	185	190	106

$[\chi^2_{0.05}$ for d.f. 3, is 7.815]

9. (a) A random sample $(x_1, x_2, x_3, x_4, x_5)$ of size 5 is drawn from a normal population with unknown mean μ.

(i) $t_1 = \dfrac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$

(ii) $t_2 = \dfrac{x_1 + x_2}{2} + x_3$

(iii) $t_3 = \dfrac{2x_1 + x_2 + \lambda x_3}{3}$

where λ is such that t_3 is an unbiased estimator of μ.

Find λ, Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3. 8

(b) Prove that the maximum likelihood estimate of the parameter α of a population having density Function $\dfrac{2}{\alpha^2}(\alpha - x)$, $0 < x < \alpha$, for a sample of unit size is 2α. 7